this note is to present briefly the more general results previously obtained in Ref. 3 and to point out several inaccurate statements in Ref. 1. The notation is that of Ref. 1.

Lighthill's solution applies for the outer layer between the spherical shock wave and the interface. It should be pointed out that only the second of the three approximations stated in Ref. 1, namely, that of constant density in the flow field between the body and the shock, is required in that analysis. If one then starts with a spherical shock wave and expresses the *general* shock conditions in terms of the shock density ratio, one obtains a problem for which Lighthill's solution is both *exact* and *unique*. The resulting body is a sphere concentric with the shock wave (for further discussion, see Ref. 4). Note that strong shock conditions are not used and that the uniqueness of the solution requires that the shock wave be spherical if the body is assumed to be a sphere. It is unfortunate that Lighthill's original paper is misleading on these two points.

The solution in the inner layer presented in Ref. 1 can be generalized[3] if one relaxes the assumption of injection normal to the body surface [Eq. (A10) of Ref. 1]. The equation to be solved is

$$\frac{1}{r^2}\frac{d^2\Psi}{dr^2} - \frac{2\Psi}{r^4} = D = \frac{\omega}{r\sin\varphi} \tag{1}$$

which is a first integral of Eq. (A12) of Ref. 1. Here D is the constant characterizing the rotationality in the inner layer. The solution of Eq. (1), subject to the conditions at the interface [Eqs. (A8) and (A9) of Ref. 1], is

$$\Psi = \frac{Dr_0^4}{30}(3\bar{r}^4 - 5\bar{r}^2 + 2\bar{r}^{-1}) + \frac{\beta_c r_0^2}{3}(\bar{r}^2 - \bar{r}^{-1}) \tag{2}$$

where $\beta_c = [dv_\varphi/d\sin\varphi]_c$ and is given from the outer layer solution by Eq. (A14) of Ref. 1. Note that the constant D is unspecified so far.

The solution permits vectored injection of the form $(v_r)_w \sim \cos\varphi$, $(v_\varphi)_w \sim \sin\varphi$, or

$$\tan\alpha = (v_\varphi)_w/(v_r)_w = A\tan\varphi \tag{3}$$

where α is the injection angle and the constant A specifies the degree of vectoring.

Using the boundary conditions at the body surface, one obtains for the rotationality constant the expression

$$D = \frac{5\beta_c}{r_0^2}\frac{[2 + \bar{R}_0^{-3} - 2A(\bar{R}_0^{-3} - 1)]}{[\bar{R}_0^{-3} + 5 - 6\bar{R}_0^2 - A(2\bar{R}_0^{-3} - 5 + 3\bar{R}_0^2)]} \tag{4}$$

and for the radial velocity at the body surface the expression

$$\frac{(v_r)_w}{\cos\varphi} = \frac{2\beta_c}{3}(\bar{R}_0^{-3} - 1) \times \left\{1 + \frac{(2 + \bar{R}_0^{-3})/(\bar{R}_0^{-3} - 1) - 2A}{(12\bar{R}_0^2 - 10 - 2\bar{R}_0^{-3})/(2\bar{R}_0^{-3} - 5 + 3\bar{R}_0^2) + 2A}\right\} \tag{5}$$

which replaces Eq. (A15) of Ref. 1. Equations (A16) and (A18) of Ref. 1 are modified similarly.

Equation (4) shows that the rotationality depends on the injection process. In fact, if A is chosen to be

$$A = (\tfrac{1}{2})(2 + \bar{R}_0^{-3})/(\bar{R}_0^{-3} - 1) \tag{6}$$

then $D = 0$, and the inner layer is irrotational. The radial velocity at the body surface then becomes simply

$$(v_r)_w = (\tfrac{2}{3})\beta_c\cos\varphi(\bar{R}_0^{-3} - 1) \tag{7}$$

It would appear from physical considerations that, if the injection process were carried out without losses, the inner layer should be irrotational. Results for shock stand-off distance and pressure distribution based on Eqs. (6) and (7) may be found in Ref. 3. The possibility of an irrotational inner layer is in contradiction to the statement in Ref. 1 that this layer must be rotational due to the shear stresses inside the interface.

A final remark is in order concerning the empirical correction for the velocity gradient β_e of Lighthill's solution. The discrepancy between theory and experiment shown in Fig. 6 of Ref. 1 is of the order that one would expect from the constant density approximation. Actually the experimental conditions do not satisfy some of the other assumptions of the analysis. The injection rate is uniform, rather than following a cosine law. The injection region subtends a half-angle of 30°, rather than the complete subsonic region as required for the validity of the analysis. Finally, there is no experimental demonstration that the injection is in fact radial, as assumed in the analysis of Ref. 1. Although the substitution of a Newtonian velocity gradient does give somewhat better agreement with experiment, such an inconsistent procedure also can give incorrect results. As an example, in Fig. A-2 of Ref. 1, the tangential velocity immediately behind the shock wave is shown to be greater than that in front of the shock wave. This is in clear violation of the conservation of tangential momentum across the shock wave.

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Author's Reply to Comment by M. Vinokur

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THE present authors were unaware of the Lockheed report by Vinokur[1] and are grateful for having it called to their attention. The analysis of Vinokur employs the same inviscid model as that of the present authors but is more general in that vectored injection is considered in his analysis.

There are only two disagreements between the authors and Vinokur. The authors' analysis was motivated by the experiments that were reported in Ref. 2. In particular, the porous material through which the coolants were injected was fabricated by a powdered metallurgy process. This process results in a random orientation of particles near the exposed surface; the gas thus is injected in a direction normal to the surface on the mean over a surface area large compared to the pore size, which, for the authors', material is on the order of 65 μ, as noted on p. 818 of Ref. 2. In view of this property of the authors' material, their analysis was restricted to radial injection, and "vectoring" cannot be a source of error between theory and experiment as suggested by Vinokur.

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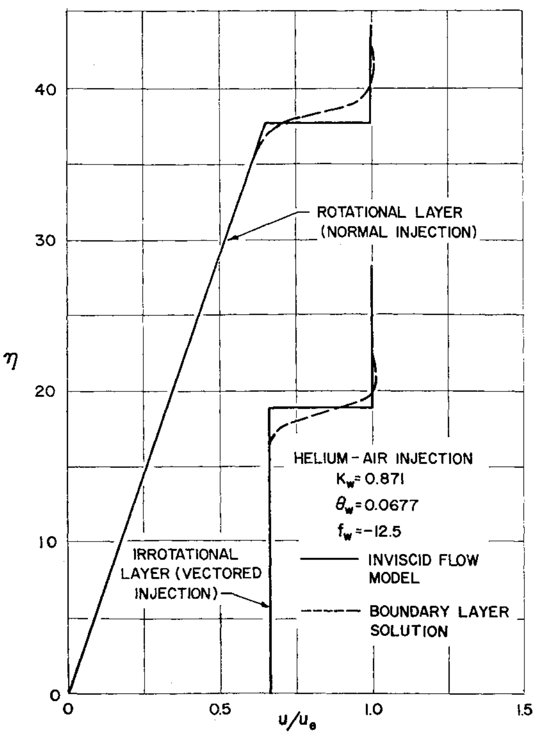

Fig. 1 Comparison of boundary layer and inviscid flow model profiles with vectored and normal injection

The second point of disagreement involves the source of the vorticity in the inner layer. Vinokur apparently associates this with "losses" in the injection process. Whether this relates to degradation of total pressure through the porous material or not is unclear to the authors. The authors' statement in a footnote on p. 825 of Ref. 2 attributes the rotationality in the inner layer to shear stresses. This point of view is substantiated by the boundary layer analysis of Fox and Libby.[3] There the relation between the boundary layer solutions and those given by the inviscid model of Vinokur and of the authors was demonstrated for "thin" inner layers. Conceptually, there is no reason why the relation between analyses including viscosity and those based on the model in question should be altered when the inner layer is thick compared to the usual shock stand-off distance.

It is, of course, true that "vectoring" will change the rotationality in the inner layer. This can be shown by an extension of the analysis of Ref. 3 to include tangential velocities at the wall; indeed the solutions of Ref. 3 can be reinterpreted to apply to vectored injection.

As an example the "free mixing" solution (solution VI of Ref. 3) can be applied to a wall flow with vectored injection so as to yield essentially irrotational flow in the inner layer. The solution identified as solution VI in Ref. 3 and tabulated in Ref. 4, which presents in extenso the numerical results given in Ref. 3, has here been extended analytically to a value of $f = f_w = -12.5$. Thus the integration yielding $\eta = \eta(\xi)$ can be carried out [cf. Eq. (29) of Ref. 3]. With the solution extended to the value of f_w just given, the velocity distributions with vectored injection can be compared to solution V of Ref. 3, which corresponds to the same normal velocity but with no slip at the wall.

Such a comparison is shown in Fig. 1 along with the corresponding velocity distributions given by the inviscid model. It will be noted, as may be expected on physical grounds, that "vectoring" reduces the thickness of the inner layer. Of significance for the present discussion is the sensibly irrotational nature of the inner layer with "vectoring." Note that the shear measured by the shear function $G \backsim f''$ is on the order of 10^{-5} at $\eta = 0$. Thus the rotationality in the inner layer depends on the tangential velocity at the wall; the relevance of the "losses" related to the injection process is not clear to the authors.

The comparison shown in Fig. 1 clearly implies that the dependence of rotationality and tangential velocity at the wall is through the viscous equations of motion that are given for thin inner layers by the boundary layer equations. Thus it appears to the authors that shear in the inner layer is the source of the rotationality.

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Comment on "Planar Librations of an Extensible Dumbbell Satellite"

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In his interesting paper,[1] Paul discussed the feasibility of damping the libration mode of an elongated satellite by means of a damper that is coupled to the libration mode through the centrifugal forces due to the libration velocity. Not unexpectedly, it is found that, since the libration mode period is long, the coupling to the damping is not very effective as regards to stopping the libration. The problem is interesting because too heavy a damper will (essentially) couple the elements rigidly and thus have no effect on the libration mode. A different damper configuration that appears to have a more effective coupling between the system modes is suggested in Fig. 1, where a simplified dumbbell is shown consisting of equal masses m connected by a weightless rod of length $2a$. The angle θ shown is the libration angle measured from local vertical. To damp the libration, a symmetrical flywheel of polar moment of inertia I may be attached at the mass center of the satellite. This flywheel may turn freely with respect to the dumbbell, except insofar as it is restrained by the viscous damper indicated in the figure. Denoting the rotation of the flywheel with respect to local vertical by φ, equations of motion of the system easily are found to be

$$\begin{aligned} \ddot{\theta} + (c/2ma^2)\dot{\theta} + 3\Omega^2\theta - (c/2ma^2)\dot{\varphi} &= 0 \\ -(c/I)\dot{\theta} + \ddot{\varphi} + (c/I)\dot{\varphi} &= 0 \end{aligned} \tag{1}$$

where dots indicate time differentiation, and Ω is the orbital circular frequency, just as in Ref. 1. Assuming solutions of the form

$$\theta = \Theta e^{\lambda t} \qquad \varphi = \Phi e^{\lambda t} \tag{2}$$

the characteristic equation of the problem is found to be

$$\lambda^4 + (c_1 + c_2)\lambda^3 + 3\Omega^2\lambda^2 + 3\Omega^2 c_2 \lambda = 0 \tag{3}$$

where $c_1 = c/2ma^2$, and $c_2 = c/I$. Denote the four roots of

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